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New Delay-Dependent Stability Conditions for Switched Time-Delay Systems

Kermani Marwen, Sakly Anis

Research Unit of Industrial Systems Study and Renewable Energy (ESIER), National Engineering School of Monastir (ENIM), Ibn El Jazzar, Skaness, 5019, Monastir, Tunisia.

kermanimarwen@gmail.com
sakly_anis@yahoo.fr

***Abstract*— The stability analysis problem is studied in this paper for continuous-time switched time-delay systems. These systems are given by delayed differential equations. Therefore, a transformation under the arrow form is employed. Indeed, by contracting an appropriated Lyapunov function and as well by using the Kotelyanski lemma associated with the M-matrix proprieties, new delay-dependent stability conditions under arbitrary switching are deduced. The obtained result allows us to avoid the search of a common Lyapunov function which is a very difficult problem. Finally, a numerical example is presented to illustrate the effectiveness of the proposed approach.**

***Keywords*— Continuous-time switched time-delay systems; M-matrix proprieties; Kotelyanski lemma; arbitrary switching**

I. INTRODUCTION

A switched system is a type of hybrid dynamical system that usually consists of a family of systems which are modelled by differential or difference equations and a rule that determines the system that is activated at certain time interval. As a special class of hybrid system, many dynamical systems can be modelled as switched systems [1–6].

On the other hand, time-delay phenomena are very common in practical systems. Indeed, switched time-delay systems have various applications in practical engineering systems such as power systems [7], power electronics [8] and networked control systems [9]. However, it is very importance to investigate switched time-delay systems. Therefore, the presence of delay makes the analysis and synthesis problems for these systems much more complicated.

Undoubtedly, stability is the first requirement for a system to work normally. Therefore, stability under arbitrary switching is fundamental in the design and analysis of switched systems [2, 5, 6, 9-14]. Indeed, this problem has been difficult and essential in researches. However, it becomes more complicated when switched time-delay systems are considered. In this framework, a common Lyapunov function for all subsystems was proved to be a necessary and sufficient condition for such systems to be asymptotically stable under arbitrary switching [6, 11-13]. But, this method is usually very difficult to apply even for continuous-time switched linear systems [2, 14]. Thus, the conservatism related to the common Lyapunov function has motivated us to look for another alternative.

In this paper, based on the construction of a common Lyapunov function and the use of the Kotelyanski lemma [15-20] combined to the $M-$matrix properties [21-23], new delay-dependent sufficient stability conditions for continuous-time switched delay systems under arbitrary switching are deduced. Noted that, this proposed method can guarantee the stability under arbitrary switching and allows us to avoid the search of a common Lyapunov function.

Within the frame of studying the stability analysis, this approach was introduced in [15, 16] for continuous-time delay systems and in our previous work [17-19] for discrete-time switched time-delay systems.

The rest of this paper is organized as follows. Section 2 formulates the problem and presents the preliminary results. The main results of this paper are presented in Section 3. Section 4 is devoted to derive new delay-independent conditions for asymptotic stability of switched system defined by differential equations. Numerical example is given to illustrate the effectiveness of the proposed approach in Section 5. Finally, we conclude in Section 6.

II. PROBLEM FORMULATION AND PRELILINAIES

A. *Problem formulation*

Consider the following switched time-delay system formed by N subsystems given in the state form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} v_i(t)\big(A_i x(t) + D_i x(t-h)\big) \\ x(t) = \phi(t) \quad t \in [-h, 0] \end{cases} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $h > 0$ is the time delay, A_i and D_i are constant matrices of appropriate dimensions for $i \in N$. $\phi(t)$ is the continuous initial function. The switching sequence is defined through a switching vector $v(t)=[v_1(t),..., v_N(t)]^T$ whose components $v_i(t)$: $\Re_+ \to M=\{0,1\}$ are an exogenous functions that depends only on the time and not on the state, they are defined through :

$$v_i(t)=\begin{cases}1 & \text{if } A_i \text{ and } D_i \text{ are actived} \\ 0 & \text{otherwise}\end{cases} \quad \text{and} \quad \sum_{i=1}^{N} v_i(t)=1 \qquad (2)$$

B. Notations and definitions

Throughout this paper, if not explicitly stated, matrices are assumed to have compatible dimensions. I is an identity matrix with appropriate dimension. Let $\Re^n$ denoted an n dimensional linear vector space over the reals $\|.\|$ stands for the Euclidean norm of vectors. For any $u=(u_i)_{1\le i\le n}$, $v=(v_i)_{1\le i\le n}\in\Re^n$ we define the scalar product of the vector u and v as: $\langle u,v\rangle=\sum_{i=1}^{n}u_i v_i$. $\lambda(M)$ denote the set of eigenvalues of matrix $M=(m_{i,j})_{1\le i,j\le n}$, M^T its transpose and M^{-1} its inverse and we denote $M^*=(m^*_{i,j})_{1\le i,j\le n}$ with $m^*_{i,j}=m_{i,j}$ if $i=j$ and $m^*_{i,j}=|m_{i,j}|$ if $i\ne j$ and $|M|=|m_{i,j}|$, $\forall i,j$.

In order to have a precise formulation of our results, we first introduce the following definitions and lemma.

Definition 1. The system (1) is said to be uniformly asymptotically stable if for any $\varepsilon>0$, there is a $\delta(\varepsilon)>0$ such that $\max_{-h\le t\le 0}\|\phi(t)\|<\delta$ implies $\|x(t,\phi)\|\le\varepsilon$, $t\ge 0$. For arbitrary switching $v(t)$, and there is also a δ' such that $\max_{-h\le t\le 0}\|\phi(t)\|<\delta'$ implies $\|x(t,\phi)\|\to 0$ as $t\to\infty$ for arbitrary switching signal.

In the next, we introduce the Kotelyanski lemma.

Kotelyanski lemma. [17] The real parts of the eigenvalues of matrix A, with non-negative off-diagonal elements, are less than a real number μ if and only if all those of matrix M, $M=\mu I_n - A$, are positive, with I_n the n identity matrix. When successive principal minors of matrix $(-A)$ are positive, Kotelyanski lemma permits to conclude on stability property of the system characterized by A .

In the following we will introduce the theorem for the $M-$matrix properties.

Theorem 1. [16] The matrix $A=(a_{ij})_{1\le i,j\le n}$ is said an $M-$matrix if the following properties are verified:

- All the eigenvalues of A have a positive real part;
- The real eigenvalues are positives;
- The principal minors of A are positive:

$$(A)\begin{pmatrix}1 & 2 & ... & j\\ 1 & 2 & ... & j\end{pmatrix}>0 \quad \forall j\in 1,...,n \qquad (3)$$

- For any positive vector $x=(x_1,...,x_n)^T$ the algebraic equations Ax have a positive solution $w=(w_1,...,w_n)^T$

Definition 2. $A=(a_{ij})_{1\le i,j\le n}$ is the opposite of an $M-$matrix if $(-A)$ is an $M-$matrix .

Remark 1. A continuous-time system characterized by a matrix A is stable if the matrix A is the opposite of an $M-$matrix . Indeed, if such condition is verified, then the principal minors of matrix $(-A)$ are positive and the Kotelyanski lemma permits to conclude on stability property of the system characterized by A .

III. MAIN RESULTS

In the following, the delay-dependent conditions will be proposed to guarantee the globally asymptotic stability of system (1) under arbitrary switching signal (2).

Theorem 2. The switched time-delay system (1) is asymptotically stable under arbitrary switching rule (2) if the matrix T_c is the opposite of an $M-$matrix .

where:

$$T_c=\max_{1\le i\le N}\left(T_{v(t)}\right) \qquad (4)$$

and:

$$T_{v(t)}=\left(A_{v(t)}+D_{v(t)}\right)^*+h\left(\left|A_{v(t)}D_{v(t)}\right|+\left|D_{v(t)}{}^2\right|\right) \qquad (5)$$

$$A_{v(t)}=\begin{bmatrix}\sum_{i=1}^{N}v_i(t)\left(a_i^{11}\right) & ... & ... & \sum_{i=1}^{N}v_i(t)\left(a_i^{1n}\right)\\ \vdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots\\ \sum_{i=1}^{N}v_i(t)\left(a_i^{n1}\right) & ... & ... & \sum_{i=1}^{N}v_i(t)\left(a_i^{nn}\right)\end{bmatrix} \qquad (6)$$

and:

$$D_{v(t)}=\begin{bmatrix}\sum_{i=1}^{N}v_i(t)\left(d_i^{11}\right) & ... & ... & \sum_{i=1}^{N}v_i(t)\left(d_i^{1n}\right)\\ \vdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots\\ \sum_{i=1}^{N}v_i(t)\left(d_i^{n1}\right) & ... & ... & \sum_{i=1}^{N}v_i(t)\left(d_i^{nn}\right)\end{bmatrix} \qquad (7)$$

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We have that a sufficient condition for asymptotic stability of system (1).

Proof [15, 16]. Let's consider the system (1) of any switching signal (2), let $(w_l>0,\forall l=1,...,n)$. By the Leibniz–Newton formula, we have $\int_{t-h}^{t} \dot{x}(s)ds = x(s) - x(s-h)$, system defined in (1) becomes:

$$x(s-h) = x(s) - \int_{t-h}^{t} \left(\sum_{i=1}^{N} v_i(t)\left(A_i x(s) + D_i x(s-h)\right) \right) ds \quad (8)$$

Then, the system defined in (1) become:

$$\dot{x}(t) = \left(\sum_{i=1}^{N} v_i(t) A_i + \sum_{i=1}^{N} v_i(t) D_i \right) x(t)$$
$$- \left(\sum_{i=1}^{N} v_i(t) A_i \right) \left(\sum_{i=1}^{N} v_i(t) D_i \right) \int_{t-h}^{t} \dot{x}(s) ds$$
$$- \left(\sum_{i=1}^{N} v_i(t) D_i \right)^2 \int_{t-h}^{t} \dot{x}(s-h) ds \text{ , } i \in \{1,...,N\} \quad (9)$$

and finally, by considering the relations (5) and (6) , system (1) can be written as follows:

$$\dot{x}(t) = \left(A_{v(t)} + D_{v(t)} \right) x(t) - \left(A_{v(t)} D_{v(t)} \right) \int_{t-h}^{t} \dot{x}(s) ds$$
$$- \left(D_{v(t)} \right)^2 \int_{t-h}^{t} \dot{x}(s-h) ds \text{ , } i \in \{1,...,N\} \quad (10)$$

Now, we define the following Lyapunov functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \quad (11)$$

where:

$$V_1(t) = \langle |x(t)|, w \rangle \quad (12)$$

$$V_2(t) = \left\langle \left| A_{v(t)} D_{v(t)} \right| \int_{h}^{0} \int_{t+\theta}^{t} |x(s)| d\theta ds, w \right\rangle \quad (13)$$

$$V_3(t) = \left\langle \left| D_{v(t)}{}^2 \right| \int_{h}^{0} \int_{t+\theta}^{t} |x(s-h)| d\theta ds, w \right\rangle \quad (14)$$

$$V_4(t) = h \left\langle \left| D_{v(t)}{}^2 \right| \int_{t-h}^{t} |x(s)| ds, w \right\rangle \quad (15)$$

it is easy to see that $V(t_0) < \infty$

Now, we are in a position to compute the derivative $V(t)$ along the trajectory of system (1), therefore:

$$\frac{dV(t)}{dt} = \frac{dV_1(t)}{dt} + \frac{dV_2(t)}{dt} + \frac{dV_3(t)}{dt} + \frac{dV_4(t)}{dt} \quad (16)$$

where:

$$\frac{dV_1(t)}{dt} = \left\langle \frac{d|x(t)|}{dt}, w \right\rangle = \left\langle \operatorname{sgn}(Dx(t)) \frac{dx(t)}{dt}, w \right\rangle \quad (17)$$

and:

$$\operatorname{sgn}(Dx(t)) = \begin{pmatrix} \operatorname{sgn}(Dx_1(t)) & & \\ & \ddots & \\ & & \operatorname{sgn}(Dx_n(t)) \end{pmatrix} \quad (18)$$

Then, we have:

$$\frac{dV_1(t)}{dt} = \left\langle \operatorname{sgn}(Dx(t)) \left(A_{v(t)} + D_{v(t)} \right) x(t) - \left(A_{v(t)} D_{v(t)} \right) \times \right.$$
$$\left. \int_{t-h}^{t} x(s) ds \left(-\left(D_{v(t)} \right)^2 \int_{t-h}^{t} x(s-h) ds \right), w \right\rangle \quad (19)$$

$$< \left\langle \left(A_{v(t)} + D_{v(t)} \right)^* |x(t)| + \left(\left| A_{v(t)} D_{v(t)} \right| \right) \int_{t-h}^{t} x(s) ds \right.$$
$$\left. + \left(\left| D_{v(t)} \right|^2 \int_{t-h}^{t} x(s-h) ds \right), w \right\rangle \quad (20)$$

and finally:

$$= \left\langle \left(A_{v(t)} + D_{v(t)} \right)^* |x(t)|, w \right\rangle + \left\langle \left(\left| A_{v(t)} D_{v(t)} \right| \right) \int_{t-h}^{t} x(s) ds, w \right\rangle$$
$$+ \left\langle \left| D_{v(t)} \right|^2 \int_{t-h}^{t} x(s-h) ds, w \right\rangle \quad (21)$$

$$\frac{dV_2(t)}{dt} = \left\langle \left(\left| A_{v(t)} D_{v(t)} \right| \right) \left(h|x(t)| - \int_{t-h}^{t} |x(s)| ds \right), w \right\rangle \quad (22)$$

$$= h \left\langle \left| A_{v(t)} D_{v(t)} \right| |x(t)|, w \right\rangle - \left\langle \left| A_{v(t)} D_{v(t)} \right| \int_{t-h}^{t} |x(s)| ds, w \right\rangle \quad (23)$$

$$\frac{dV_3(t)}{dt} = \left\langle \left(\left| D_{v(t)}{}^2 \right| \right) \left(h|x(t-h)| - \int_{t-h}^{t} |x(s-h)| ds \right), w \right\rangle \quad (24)$$

$$= \left\langle h \left| D_{v(t)}{}^2 \right| |x(t-h)|, w \right\rangle - \left\langle \left| D_{v(t)}{}^2 \right| \int_{t-h}^{t} |x(s-h)| ds, w \right\rangle \quad (25)$$

$$\frac{dV_4(t)}{dt} = \left\langle h \left| D_{v(t)}{}^2 \right| \left(|x(t)| - |x(t-h)| \right), w \right\rangle \quad (26)$$

$$= \left\langle h \left| D_{v(t)}{}^2 \right| |x(t)|, w \right\rangle - \left\langle h \left| D_{v(t)}{}^2 \right| |x(t-h)|, w \right\rangle \quad (27)$$

Finally, by adding equations (21), (23), (25) and (26) we obtain:

$$\frac{dV(t)}{dt} < \left\langle \left(A_{v(t)} + D_{v(t)} \right)^* |x(t)|, w \right\rangle + \left\langle h \left| A_{v(t)} D_{v(t)} \right| |x(t)|, w \right\rangle$$
$$+ \left\langle h \left| D_{v(t)}{}^2 \right| |x(t)|, w \right\rangle = \left\langle T_{v(t)} |x(t)|, w \right\rangle \quad (28)$$

and finally:

$$\frac{dV(t)}{dt} < \langle T_c |x(t)|, w \rangle \quad (29)$$

where T_c is defined in (4).

Knowing that:

$$\langle T_c |x(t)|, w \rangle = \langle T_c{}^T w, |x(t)| \rangle \quad (30)$$

On the other hand, we suppose that T_c is the opposite of an $M-$matrix , by according to the proprieties of the $M-$matrix , we can find a vector $\rho \in \Re_+^{*n} \left(\rho_l \in \Re_+^* \ \ l = 1,...,n \right)$ satisfying the next relation $\left(T_c\right)^T w = -\rho, \forall w \in \Re_+^{*n}$, so, we can establish the following relation:

$$\left\langle (T_c) |x(t)|, w \right\rangle = \left\langle (T_c)^T w, |x(k)| \right\rangle = \langle -\rho, |x(t)| \rangle \quad (31)$$

Now, taking into account (31), then relation (29) becomes:

$$\frac{dV(t)}{dt} < \langle -\rho, |x(t)| \rangle = -\sum_{l=1}^{n} \rho_l |x(t)| < 0 \qquad (32)$$

This completes the proof of Theorem 2.

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IV. APPLICATION TO SWITCHED SYSTEMS DEFINED BY DIFFERENTIAL EQUATIONS

In this section, we will applied the previous obtained results to switched systems modelled by the following functional linear delay differential equation:

$$y^n(t) + \sum_{i=1}^{N} v_i(t)\left(\sum_{j=1}^{n-1} a_i^j y^{(j)}(t)\right) + \sum_{i=1}^{N} v_i(t)\left(\sum_{j=1}^{n-1} d_i^j y^{(j)}(t-h)\right) = 0, \ h > 0 \qquad (33)$$

The presence of delay renders the stability study of this problem very difficult. However to solve it, the proposed solution consists to use the following matrix representation [15-19]:

$$x_{j+1}(t) = \frac{dy^{(j)}}{dt^{(j)}}, \ j = 1,...,n-1 \qquad (34)$$

By substituting relation (34) in equation (33), we obtain:

$$\begin{cases} \dot{x}_j(t) = x_{j+1}(t) \\ \dot{x}_n(t) = -\sum_{i=1}^{N} v_i(t)\left(\sum_{j=0}^{n-1} a_i^j x_{j+1}(t) + \sum_{j=0}^{n-1} d_i^j x_{j+1}(t-h)\right) \end{cases} \qquad (35)$$

or in matrix form, we obtain the following state representation:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} v_i(t)\left(A_i x(t) + D_i x(t-h)\right) \\ x(t) = \phi(t) \quad t \in [-h, 0] \end{cases} \qquad (36)$$

where $x(t)$ is the state vector, whose these components are $x_j(t)$, $j = 1,...,n-1$, $v_i(t)$ is the switching signal given in (2), and matrices A_i and D_i are given as following [17]:

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ -a_i^0 & -a_i^1 & \cdots & -a_i^{n-1} \end{bmatrix} \qquad (37)$$

$$D_i = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ -d_i^0 & -d_i^1 & \cdots & -d_i^{n-1} \end{bmatrix} \qquad (38)$$

where a_i^j is a coefficient of the instantaneous characteristic polynomial $P_i(\lambda)$ of the matrix A_i given by:

$$P_i(\lambda) = \lambda^n + \sum_{q=0}^{n-1} a_i^q \lambda^q \qquad (39)$$

and d_i^j is a coefficient of the instantaneous characteristic polynomial $Q_i(\lambda)$ of the matrix D_i defined such as:

$$Q_i(\lambda) = \sum_{q=0}^{n-1} d_i^q \lambda^q \qquad (40)$$

Now, we will introduced the polynomial ψ_i [16], it is given as follows:

$$\psi_i(\lambda) = Q_i(\lambda)\lambda + d_i^{n-1} P_i(\lambda) = \sum_{q=0}^{n-1} \zeta_i^q \lambda^q \qquad (41)$$

where the parameters ζ_i^q, $q = 0,...,n-1$ are given by:

$$\begin{cases} \zeta_i^0 = d_i^{n-1} a_i^0 \\ \zeta_i^q = d_i^{n-1} a_i^q - d_i^{q-1}, \ q = 1,...,n-2 \\ \zeta_i^{n-1} = d_i^{n-1} a_i^{n-1} \end{cases} \qquad (42)$$

Therefore, to simplifier the application of the Kotelyanski lemma a change to base of the system (36) into the arrow matrix form will be considered.
Leads the new state vector [18]:

$$z(t) = Px(t) \qquad (43)$$

where:

$$P = \begin{bmatrix} 1 & 1 & \cdots & 1 & 0 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & 0 \\ (\alpha_1)^2 & (\alpha_2)^2 & \cdots & (\alpha_{n-1})^2 & \vdots \\ \vdots & \vdots & \cdots & \vdots & 0 \\ (\alpha_1)^{n-1} & (\alpha_2)^{n-1} & \cdots & (\alpha_{n-1})^{n-1} & 1 \end{bmatrix} \qquad (44)$$

and $\alpha_j \ (j = 1,...n-1)$, $\alpha_j \neq \alpha_q \ \forall j \neq q$, $i = 1,...,N$ are free real parameters, that can be chosen arbitrary.
Leads to the following space representation:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{N} v_i(t)\left(M_i z(t) + N_i z(t-h)\right) \\ z(t) = P\phi(t) \quad t \in [-h, 0] \end{cases} \qquad (45)$$

where M_i and N_i are in the arrow form [15-19] given as follows:

$$M_i = P^{-1} A_i P = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 & \beta_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \alpha_{n-1} & \beta_{n-1} \\ \gamma_i^1 & \cdots & \cdots & \gamma_i^{n-1} & \gamma_i^n \end{bmatrix} \qquad (46)$$

$$N_i = P^{-1} D_i P = \begin{bmatrix} 0_{n-1,n-1} \cdots & & 0_{n-1,1} \\ \delta_i^1 \cdots & \delta_i^{n-1} & \delta_i^n \end{bmatrix} \qquad (47)$$

with:

$$\beta_j = \prod_{\substack{q=1\\q\neq j}}^{n-1}\left(\alpha_j-\alpha_q\right)^{-1},\ \forall\ j=1,2,...,n-1 \tag{48}$$

Then, the elements of the matrices M_i are:

Taking into account the previous relations, the matrices T_i $i=1,...N$ will define as following:

$$\begin{cases}\gamma_i^j = -P_i\left(\alpha_j\right)\ ,\forall\ j=1,...,n-1\\ \gamma_i^n = -a_i^{n-1}-\sum_{j=1}^{n-1}\alpha_j\end{cases} \tag{49}$$

and the elements of the matrices N_i are:

$$\begin{cases}\delta_i^j = -Q_i\left(\alpha_j\right),\forall\ j=1,...,n-1\\ \delta_i^n=-d_i^{n-1}\end{cases} \tag{50}$$

$$T_i=\begin{bmatrix}\alpha_1 & 0 & \cdots & 0 & |\beta_1|\\ 0 & \ddots & \ddots & \vdots & \vdots\\ \vdots & \ddots & \ddots & 0 & \vdots\\ 0 & \cdots & 0 & \alpha_{n-1} & |\beta_{n-1}|\\ t_i^1 & \cdots & \cdots & t_i^{n-1} & t_i^n\end{bmatrix} \tag{51}$$

and the matrix $T_{v(t)}$ is given by:

$$T_{v(t)}=\begin{bmatrix}\alpha_1 & 0 & \cdots & 0 & |\beta_1|\\ 0 & \ddots & \ddots & \vdots & \vdots\\ \vdots & \ddots & \ddots & 0 & \vdots\\ 0 & \cdots & 0 & \alpha_{n-1} & |\beta_{n-1}|\\ \sum_{i=1}^{N}v_i(t)t_i^1 & \cdots & \cdots & \sum_{i=1}^{N}v_i(t)t_i^{n-1} & \sum_{i=1}^{N}v_i(t)t_i^n\end{bmatrix} \tag{52}$$

where:

$$\begin{cases}t_i^j=\left|\gamma_i^j+\delta_i^j\right|+h\left(\left|\psi_i\left(\alpha_j\right)\right|+\left|\delta_i^j d_i^{n-1}\right|\right), j=1,...,n\text{-}1\\ t_i^n=\gamma_i^n+\delta_i^n+h\left(\left|a_i^{n-1}d_i^{n-1}\right|+\left(d_i^{n-1}\right)^2\right)\end{cases} \tag{53}$$

Now, we can deduce the following theorem.

Theorem 3. The system (36) is globally asymptotically stable under arbitrary switching rule (2) if there exist $\alpha_j<0\ \ (j=1,...,n-1)$, $\alpha_j\neq\alpha_q$, $\forall\ j\neq q$, satisfying the following condition:

$$-\bar{t}^n+\sum_{j=1}^{n-1}\bar{t}^j\left|\beta_j\right|\alpha_j^{-1}>0 \tag{54}$$

where:

$$\begin{cases}\bar{t}^n=\max\left(t_i^n\right),\ i=1,...N\\ \bar{t}^j=\max\left(t_i^j\right),\ j=1,...,n-1,\ i=1,...N\end{cases} \tag{55}$$

Proof. For an arbitrary choice $\alpha_j<0\ \ (j=1,...,n-1)$, $\alpha_j\neq\alpha_q$, $\forall\ j\neq q$ and according to Kotelyanski lemma. In this case, it suffices to verify that the matrix $T_c=\max_{1\leq i\leq N}\left(T_{v(t)}\right)$ is the opposite of an $M-$matrix. These conditions require having the all the principal minors are positive. The $n-1$ first conditions are checked because the α_j are negative, however the last condition yields to: $\det\left(-T_c\right)=\chi\prod_{j=1}^{n-1}\alpha_j>0$. where $\chi=\bar{t}^n-\sum_{j=1}^{n-1}\bar{t}^j\left|\beta_j\right|\alpha_j^{-1}$. It comes $\chi<0$, then condition (54) are verifies.

V. NUMERICAL EXAMPLE

Consider the following continuous-time switched time-delay system:

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^{2}v_i(t)\left(A_i x(t)+D_i x(t-h)\right)\\ x(t)=\phi(t)\quad t\in[-h,0]\end{cases}$$

where:

$A_1=\begin{bmatrix}0 & 1\\ -1 & -10\end{bmatrix}$, $A_2=\begin{bmatrix}0 & 1\\ -0.2 & -10\end{bmatrix}$, $D_1=\begin{bmatrix}0 & 0\\ -2 & 0\end{bmatrix}$, $D_2=\begin{bmatrix}0 & 0\\ -1 & 0\end{bmatrix}$ and $h>0$ is the time delay.

By using the algebraic properties given in (44), (45), (46), (47), (48), (49) and (50), the matrices M_1, M_2, N_1 and N_2 in the arrow form are the following:

$$M_1=\begin{bmatrix}\alpha & \beta\\ -\left(\alpha^2+10\alpha+1\right) & -10-\alpha\end{bmatrix},$$

$$M_2=\begin{bmatrix}\alpha & \beta\\ -\left(\alpha^2+10\alpha+0.2\right) & -(10+\alpha)\end{bmatrix},$$

$N_1=\begin{bmatrix}0 & 0\\ -2 & 0\end{bmatrix}$ and $N_2=\begin{bmatrix}0 & 0\\ -1 & 0\end{bmatrix}$.

For particular choose, $\alpha=-1$, $\beta=1$ then the stability conditions of Theorem 3 are verified as following : $-\max(-9,-9)-\max(8.3,6)=0.7>0$.

Now, by choosing the final time $t_f=20s$, the switched time $t_1=10s$ and the time delay $h=2s$. The simulation results are shown in Fig. 1 and Fig. 2 where the initial function $\phi(t)=\begin{bmatrix}-2 & 1\end{bmatrix}^T$. Fig. 2 shows the state responses, the state trajectories are depicted in Fig. 2, which show the stability of the system given in the example.

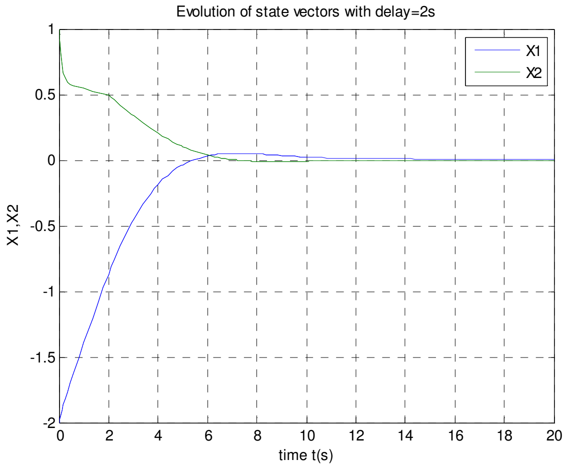

Fig. 3. The state responses of the system given in the example

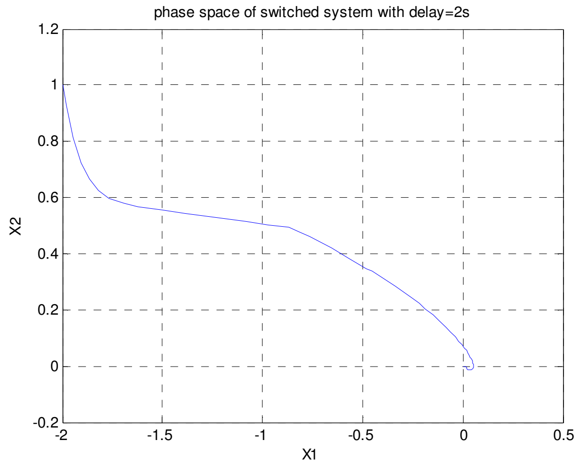

Fig. 2. Trajectory response of the system given in the example

VI. CONCLUSION

In this paper, new delay-dependent stability conditions under arbitrary switching for continuous-time switched time delay systems are established. These stability conditions were derived from the contraction of an appropriate Lyapunov function associated with the application of the Kotelyanski lemma and the $M-$matrix properties. Compared with the existing results, the benefit of this method is that, it can avoid the research of a common Lyapunov function which is usually very difficult, or even not possible. Simulation results have been presented to illustrate the effectiveness of the developed method.

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